MATLAB PROJECT 3

Please include this page in your Group file, as a front page. Type in the group number and the names of all members WHO PARTICIPATED in this project.

GROUP # 15 \_

FIRST & LAST NAMES (UFID numbers are NOT required):

1. Drew Goldsmith
2. Ricardo Canelon
3. Paola Solari
4. Carlo Romo
5. Guillermo Canelon

**By signing your names above, each of you had confirmed that you did the work and agree with the work submitted**.

# Exercise 1 (Carlo Romo)

diary on

**% Exercise 1**

format compact

type subspace

function [] = subspace(A, B)

m=size(A,1);

n=size(B,1);

if m==n

sprintf('Col A and Col B are subspaces of R^%i\n',m)

else

sprintf('Col A and Col B are subspaces of different spaces')

return

end

k = rank(A);

p = rank(B);

At = transpose(A);

Bt = transpose(B);

fprintf('dim of Col A is k = %i\n',k);

fprintf('dim of Col B is p = %i\n',p);

if k~=p

fprintf('k ~ = p, the dimensions of Col A and Col B are different\n');

else

if isequal(rref(At), rref(Bt)) == 1

fprintf('Col A = Col B\n');

else

fprintf('k = p, the dimensions of Col A and Col B are the same, but Col A ~ = Col B\n');

end

end

if k==m

fprintf('k = m (%i = %i) Col A is all R^%i\n', k, m, m);

else

fprintf('k != m (%i != %i) Col A is not all R^%i\n', k, m, m);

end

if p==m

fprintf('p = n (%i = %i) Col B is all R^%i\n', p, n, n);

else

fprintf('p != n (%i != %i) Col B is not all R^%i\n', p, n, n);

end

end

A = [2 -4 -2 3; 6 -9 -5 8; 2 -7 -3 9; 4 -2 -2 -1; -6 3 3 4]

A =

2 -4 -2 3

6 -9 -5 8

2 -7 -3 9

4 -2 -2 -1

-6 3 3 4

B = rref(A)

B =

1.0000 0 -0.3333 0

0 1.0000 0.3333 0

0 0 0 1.0000

0 0 0 0

0 0 0 0

subspace(A, B)

ans =

'Col A and Col B are subspaces of R^5

'

dim of Col A is k = 3

dim of Col B is p = 3

k = p, the dimensions of Col A and Col B are the same, but Col A ~ = Col B

k != m (3 != 5) Col A is not all R^5

p != n (3 != 5) Col B is not all R^5

A= magic(4), B = eye(4)

A =

16 2 3 13

5 11 10 8

9 7 6 12

4 14 15 1

B =

1 0 0 0

0 1 0 0

0 0 1 0

0 0 0 1

subspace(A, B)

ans =

'Col A and Col B are subspaces of R^4

'

dim of Col A is k = 3

dim of Col B is p = 4

k ~ = p, the dimensions of Col A and Col B are different

k != m (3 != 4) Col A is not all R^4

p = n (4 = 4) Col B is all R^4

A= magic(4), B = eye(3)

A =

16 2 3 13

5 11 10 8

9 7 6 12

4 14 15 1

B =

1 0 0

0 1 0

0 0 1

subspace(A, B)

ans =

'Col A and Col B are subspaces of different spaces'

A = magic(5), B=eye(5)

A =

17 24 1 8 15

23 5 7 14 16

4 6 13 20 22

10 12 19 21 3

11 18 25 2 9

B =

1 0 0 0 0

0 1 0 0 0

0 0 1 0 0

0 0 0 1 0

0 0 0 0 1

subspace(A, B)

ans =

'Col A and Col B are subspaces of R^5

'

dim of Col A is k = 5

dim of Col B is p = 5

Col A = Col B

k = m (5 = 5) Col A is all R^5

p = n (5 = 5) Col B is all R^5

**%Elementary row operations will not change the column space of a matrix because it does not change the linear dependence or independence between columns, only rows.**

diary off

# Exercise 2 (Drew Goldsmith)

diary on

format compact

**% Exercise2**

A=magic(4)

A =

16 2 3 13

5 11 10 8

9 7 6 12

4 14 15 1

type shrink

function B = shrink(A)

[~,pivot]=rref(A);

B=A(:,pivot);

end

[~,pivot]=rref(A)

pivot =

1 2 3

B=A(:,pivot)

B =

16 2 3

5 11 10

9 7 6

4 14 15

**%the first command "[~,pivot]=rref(A)" outputs the columns that are pivot columns**

**%the second command "B=A(:,pivot)" creates a new Matrix 'B' that contains only the pivot columns**

type basis

function B=basis(A)

**%BASIS Summary of this function goes here**

**% Detailed explanation goes here**

m=size(A,1);

A=shrink(A);

sprintf('a basis for Col A is \n')

B=A

D=[B,magic(m)];

D=shrink(D);

[rows,~]=size(A);

if rank(B)==rows,

sprintf('A basis for R^%i is \n',m)

else

if rank(D)==rows,

sprintf('A basis for R^%i is \n',m)

B=D

else

disp('What? It is not a basis!?')

end

end

end

diary on

A=[1,0;0,0;0,0;0,1]

A =

1 0

0 0

0 0

0 1

**%(a)**

A=[1,0;0,0;0,0;0,1]

A =

1 0

0 0

0 0

0 1

B=basis(A)

ans =

'a basis for Col A is

'

B =

1 0

0 0

0 0

0 1

ans =

'A basis for R^4 is

'

B =

1 0 16 2

0 0 5 11

0 0 9 7

0 1 4 14

B =

1 0 16 2

0 0 5 11

0 0 9 7

0 1 4 14

**%(b)**

A=[2,0;4,0;1,0;0,0]

A =

2 0

4 0

1 0

0 0

B=basis(A)

ans =

'a basis for Col A is

'

B =

2

4

1

0

ans =

'A basis for R^4 is

'

B =

2 16 2 3

4 5 11 10

1 9 7 6

0 4 14 15

B =

2 16 2 3

4 5 11 10

1 9 7 6

0 4 14 15

**%(c)**

A=magic(3)

A =

8 1 6

3 5 7

4 9 2

B=basis(A)

ans =

'a basis for Col A is

'

B =

8 1 6

3 5 7

4 9 2

ans =

'A basis for R^3 is

'

B =

8 1 6

3 5 7

4 9 2

**%(d)**

A=magic(6)

A =

35 1 6 26 19 24

3 32 7 21 23 25

31 9 2 22 27 20

8 28 33 17 10 15

30 5 34 12 14 16

4 36 29 13 18 11

B=basis(A)

ans =

'a basis for Col A is

'

B =

35 1 6 26 19

3 32 7 21 23

31 9 2 22 27

8 28 33 17 10

30 5 34 12 14

4 36 29 13 18

What? It is not a basis!?

B =

35 1 6 26 19

3 32 7 21 23

31 9 2 22 27

8 28 33 17 10

30 5 34 12 14

4 36 29 13 18

diary off

# Exercise 3 (Ricardo Canelon)

**% Exercise 3**

diary on

**% Function CloseToZeroRoundOff**

type closetozeroroundoff

function B=closetozeroroundoff(A)

[m,n]=size(A);

for i=1:m

for j=1:n

if abs(A(i,j)) < 10^(-7)

A(i,j) = 0;

end

end

end

B=A;

**% Function PolySpace**

type polyspace

function P = polyspace(B,Q,r)

format rat

u = sym2poly(B(1));

n = length(u);

C = zeros(n);

for i = 1:n

C(:, i) = transpose(sym2poly(B(i)));

end

P = closetozeroroundoff(C);

if rank(P) ~= n

disp('The polynomials in B do not form a basis for P')

disp('The reduced echelon form of P is')

A = rref(P);

P = A;

else

disp('The polynomials in B form a basis for P')

disp('‘The coordinates of the polynomial Q with respect to the basis P are')

y = transpose(inv(P) \* transpose(sym2poly(Q)));

y = closetozeroroundoff(y)

disp('The polynomial in standard coordinates that corresponds to r is')

q = (P \* r);

q = transpose(closetozeroroundoff(q));

R = poly2sym(q)

end

end

**% (a)**

syms x

B = [x^3+3\*x^2,10^(-8)\*x^3+x,10^(-8)\*x^3+4\*x^2+x,x^3+x]

B = [ x^3 + 3\*x^2, x^3/100000000 + x, x^3/100000000 + 4\*x^2 + x, x^3 + x]

Q = 10^(-8)\*x^3+x^2+6\*x

Q = x^3/100000000 + x^2 + 6\*x

r = [2;-3;1;0]

r =

2

-3

1

0

P = polyspace(B,Q,r)

The polynomials in B do not form a basis for P

The reduced echelon form of P is

P =

1 0 0 1

0 1 0 7/4

0 0 1 -3/4

0 0 0 0

**% For P to be a basis for R^n, P must have n linearly independent vectors as columns. Given the fact that matrix P is not row equivalent to the identity matrix as shown by our result, we can say that the columns of P are not a linearly independent set and therefore cannot span R^n. In other words, P cannot be a basis for R^n since its columns do not form a linearly independent set.**

**% (b)**

B = [x^3-1,10^(-8)\*x^3+2\*x^2,10^(-8)\*x^3+x,x^3+x]

B = [ x^3 - 1, x^3/100000000 + 2\*x^2, x^3/100000000 + x, x^3 + x]

P = polyspace(B,Q,r)

The polynomials in B form a basis for P

‘The coordinates of the polynomial Q with respect to the basis P are

y = 0 1/2 6 0

The corresponding polynomial in standard coordinates that corresponds to r is

R = 2\*x^3 - 6\*x^2 + x – 2

P =

1 0 0 1

0 2 0 0

0 0 1 1

-1 0 0 0

**% (c)**

B = [x^4+x^3+x^2+1,10^(-8)\*x^4+x^3+x^2+x+1,10^(-8)\*x^4+x^2+x+1, 10^(-8)\*x^4+x+1,10^(-8)\*x^4+1]

B = [ x^4 + x^3 + x^2 + 1, x^4/100000000 + x^3 + x^2 + x + 1, x^4/100000000 + x^2 + x + 1, x^4/100000000 + x + 1, x^4/100000000 + 1]

Q=x^4-1

Q = x^4 – 1

r=diag(magic(5))

r =

17

5

13

21

9

P = polyspace(B,Q,r)

The polynomials in B form a basis for P

‘The coordinates of the polynomial Q with respect to the basis P are

y = 1 -1 0 1 -2

The polynomial in standard coordinates that corresponds to r is

R = 17\*x^4 + 22\*x^3 + 35\*x^2 + 39\*x + 65

P =

1 0 0 0 0

1 1 0 0 0

1 1 1 0 0

0 1 1 1 0

1 1 1 1 1

diary off

# Exercise 4 (Guillermo Canelon)

**%Exercise 4**

diary on

type reimsum

function [T, I] = reimsum(P, a, b, n)

N = length(n);

j = 1 : N;

h = (b - a) ./ n(j);

for i = 1 : N

    for k = 1 : n(i)

        x(k) = a + (k - 1) \* h(i);

        y(k) = a + k \* h(i);

    end

    z = 1/2 \* (x+y);

    c(i) = h(i) \* sum(polyval(sym2poly(P),x));

    f(i) = h(i) \* sum(polyval(sym2poly(P),y));

    d(i) = h(i) \* sum(polyval(sym2poly(P),z));

end

d = closetozeroroundoff(d);

A = [n',c',d',f'];

T = array2table(A,'VariableNames',{'n','Left','Middle','Right'});

I = double(int(P, a, b));

format long

**%(a)**

P = 2\*x^4 + 4\*x^2 - 1

P =

2\*x^4 + 4\*x^2 - 1

a = -1;

b = 1;

n = [1:10];

I = double(int(P,a,b));

reimsum(P,a,b,n)

ans =

               Left                Middle              Right

     1                  10                   -2                  10

     2                   4                 0.25                   4

     3    2.62551440329218    0.897119341563786    2.62551440329218

     4               2.125             1.140625               2.125

     5             1.88992              1.25632             1.88992

     6    1.76131687242798     1.31995884773663    1.76131687242798

     7    1.68346522282382     1.35860058309038    1.68346522282382

     8           1.6328125         1.3837890625           1.6328125

     9     1.5980287557791     1.40110755474267     1.5980287557791

    10             1.57312              1.41352             1.57312

I

I =

   1.466666666666667

n = [1,5,10,100,1000,10000];

reimsum(P,a,b,n)

ans =

  6×4 <a href="matlab:helpPopup table" style="font-weight:bold">table</a>

    <strong>  n  </strong><strong>          Left      </strong><strong>         Middle     </strong><strong>         Right      </strong>

    <strong>\_\_\_\_\_</strong>    <strong>\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_</strong>    <strong>\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_</strong>    <strong>\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_</strong>

        1                  10                  -2                  10

        5             1.88992             1.25632             1.88992

       10             1.57312             1.41352             1.57312

      100         1.467733312         1.466133352         1.467733312

     1000     1.4666773333312     1.4666613333352     1.4666773333312

    10000    1.46666677333333    1.46666661333333    1.46666677333333

I

I =

   1.466666666666667

a = -10;

b = 10;

n = [1:10];

reimsum(P,a,b,n)

ans =

                 Left            Middle            Right

     1              407980                 -20              407980

     2              203980               26980              203980

     3    139864.773662551    55025.2674897119    139864.773662551

     4              115480             66542.5              115480

     5              103852               72172              103852

     6    97445.0205761317    75309.2181069959    97445.0205761317

     7    93551.0120783007    77227.8134110787    93551.0120783007

     8            91011.25         78483.90625            91011.25

     9    89264.3570593913    79350.0147335264    89264.3570593914

    10               88012               79972               88012

I

I =

  8.264666666666667e+04

a = -10;

b = 10;

n = [1,5,10,100,1000,10000];

reimsum(P,a,b,n)

ans =

                  Left                Middle              Right

        1              407980                 -20              407980

        5              103852               72172              103852

       10               88012               79972               88012

      100          82700.5312    82619.7352000001          82700.5312

     1000    82647.2053331201      82646.39733352    82647.2053331201

    10000    82646.6720533334    82646.6639733334    82646.6720533334

I

I =

   8.264666666666667e+04

**% Out of the three columns, the values in the ‘Middle’ section provide a better approximation for the integral**

**% (b)**

P = x^3 - 2\*x

P = x^3 - 2\*x

a = -1;

b = 1;

n = [1:10];

reimsum(P,a,b,n)

ans =

              Left           Middle            Right

     1                    2    0                         -2

     2                    1    0                         -1

     3    0.666666666666667    0         -0.666666666666667

     4                  0.5    0                       -0.5

     5                  0.4    0                       -0.4

     6    0.333333333333334    0         -0.333333333333333

     7    0.285714285714286    0         -0.285714285714285

     8                 0.25    0                      -0.25

     9    0.222222222222222    0         -0.222222222222222

    10                  0.2    0                       -0.2

I = double(int(P,a,b));

I

I =

     0

a = -1;

b = 1;

n = [1,5,10,100,1000,10000];

reimsum(P,a,b,n)

ans =

               Left               Middle            Right

        1                       2    0                            -2

        5                     0.4    0                          -0.4

       10                     0.2    0                          -0.2

      100      0.0199999999999999    0           -0.0200000000000001

     1000     0.00199999999999974    0          -0.00200000000000036

    10000    0.000200000000000055    0         -0.000199999999999922

I

I =

     0

a = -10;

b = 10;

n = [1:10];

reimsum(P,a,b,n)

ans =

              Left           Middle            Right

     1               -19600    0                    19600

     2                -9800    0                     9800

     3    -6533.33333333333    0         6533.33333333333

     4                -4900    0                     4900

     5                -3920    0                     3920

     6    -3266.66666666667    0         3266.66666666667

     7                -2800    0                     2800

     8                -2450    0                     2450

     9    -2177.77777777778    0         2177.77777777778

    10                -1960    0                     1960

I

I =

     0

a = -10;

b = 10;

n = [1,5,10,100,1000,10000];

reimsum(P,a,b,n)

ans =

               Left           Middle            Right

        1               -19600    0                    19600

        5                -3920    0                     3920

       10                -1960    0                     1960

      100    -195.999999999999    0         196.000000000001

     1000    -19.5999999999999    0         19.6000000000001

    10000    -1.95999999999928    0         1.96000000000072

**% Out of the three columns, the values in the ‘Middle’ section provide a better approximation for the integral**

# Exercise 5 (Paola Solari)

**% Exercise 5**

**% Polint Function**

type polint

function B = polint(P)

u = sym2poly(P);

a = size(u);

b = a(2);

y = 5;

for i = 1:b

u(1,i) = u(1,i)/(b+1-i);

end

L = [u y];

B = poly2sym(L);

end

syms x

P = [6\*x^5 + 5\*x^4 + 4\*x^3 + 3\*x^2 + 2\*x + 6]

P = 6\*x^5 + 5\*x^4 + 4\*x^3 + 3\*x^2 + 2\*x + 6

B = polint(P)

B = x^6 + x^5 + x^4 + x^3 + x^2 + 6\*x + 5

int(P)+5

ans = x^6 + x^5 + x^4 + x^3 + x^2 + 6\*x + 5

P = [x^6 - x^4 + 3\*x^2 + 1]

P = x^6 - x^4 + 3\*x^2 + 1

B = polint(P)

B = x^7/7 - x^5/5 + x^3 + x + 5

int(P)+5

ans = x^7/7 - x^5/5 + x^3 + x + 5

diary off

# Exercise 6 (Paola Solari)

**% Exercise 6**

diary on

type markov

function q = markov(P, x0)

**%MARKOV Summary of this function goes here**

**% Detailed explanation goes here**

**% Step 1: Check if Matrix P is stochastic**

[m,n] = size(P);

z = 0;

for i = 1:n

s(i) = sum(P(:,i));

for j = 1:m

if P(j,i)<=0

z=1;

break

end

end

end

if m ~= n || sum(s) ~= n || z == 1

disp('P is not stochastic matrix')

q=[];

return

end

**% Step 2: Find steady-state vector q**

Q = null (P-eye(n), 'r');

c = sum(Q);

q = (1/c) \* Q;

counter = 0;

**% Step 3: Verify that the Markov chain converges to q**

while norm(x0-q) >= 10e-7

x1 = P\*x0;

x0 = x1;

counter = counter + 1;

end

xk = x1

k = counter

end

P = [.6 .3; .5 .7]

P =

0.6000 0.3000

0.5000 0.7000

x0 = [.4;.6]

x0 =

0.4000

0.6000

q = markov (P, x0)

P is not stochastic matrix

q =

[]

**% (b)**

P = [.5 .3; .5 .7]

P =

0.5000 0.3000

0.5000 0.7000

q = markov (P, x0)

xk =

0.3750

0.6250

k =

7

q =

0.3750

0.6250

**% xk is the same vector as vector q**

**%(c)**

P = [.9 .2; .1 .8]

P =

0.9000 0.2000

0.1000 0.8000

x0 = [.12; .88]

x0 =

0.1200

0.8800

q = markov (P, x0)

xk =

0.6667

0.3333

k =

39

q =

0.6667

0.3333

**% (d)**

x0 = [.14; .86]

x0 =

0.1400

0.8600

q = markov (P, x0)

xk =

0.6667

0.3333

k =

38

q =

0.6667

0.3333

x0 = [.86; .14]

x0 =

0.8600

0.1400

q = markov (P, x0)

xk =

0.6667

0.3333

k =

36

q =

0.6667

0.3333

**% The output vectors q for part (d) are the same as the output q for part (c). Therefore, we can say that the value of initial vector x) does not have an effect on the steady-state vector q. Nonetheless, the number of iterations k did change, so the initial vector x0 has an effect on the steady-state vector q.**

**% (e)**

P = [.90 .01 .09; .01 .90 .01; .09 .09 .90]

P =

0.9000 0.0100 0.0900

0.0100 0.9000 0.0100

0.0900 0.0900 0.9000

x0= [.5;.3;.2]

x0 =

0.5000

0.3000

0.2000

q = markov (P, x0)

xk =

0.4354

0.0909

0.4737

k =

109

q =

0.4354

0.0909

0.4737

diary off